Logics for Qualitative Inductive Generalization*

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Dedicated to Newton da Costa
in honour of his work,
in gratitude for his friendship.

Abstract

The paper contains a survey of (mainly unpublished) adaptive logics of inductive generalization. These defeasible logics are precise formulations of certain methods. Some attention is also paid to ways of handling background knowledge, introducing mere conjectures, and the research guiding capabilities of the logics.

1 Aim of this paper

This paper describes a variety of adaptive logics of inductive generalization. By a logic I obviously mean a mapping $\mathbf{L}\colon \wp(\mathcal{W}) \rightarrow \wp(\mathcal{W})$, in which $\mathcal{W}$ is the set of closed formulas of the standard predicative language $\mathcal{L}$. There is a further requirement for $\mathbf{L}$ to be a logic: the selection of $Cn_{\mathbf{L}}(\Gamma)$ must be justified by *logical* means. Moreover, if the logic is to be a formal one, $Cn_{\mathbf{L}}(\Gamma)$ must be selected in view of Γ by a *formal* criterion.

The logics are not a contribution to the tradition of Carnapian inductive logic—see for example [14, Ch. 4]. They are logics of inductive generalization in the most straightforward sense of the term, logics that from a set of empirical data and possibly a set of background generalizations lead to a set of generalizations and their consequences. They operate in a purely qualitative way, first because of the restriction to the language, which prevents one to express statistical hypotheses, and next because the logics proceed in terms of the *types* of data that are available, while the *number* of data of each type plays no role. The logics have a proof theory and a semantics. They are characterized in a formally decent way, their metatheory may be phrased in precise terms,[1] and, most importantly, they aim at explicating people's actual reasoning towards inductive generalizations.

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[1]The metatheory cannot be included here. It is available partly in [6] and more generally in [9], of which seven chapters are available on the internet at the moment I am writing this.

The adaptive logics are formally stringent formulations of inductive methods, whence there should be many. The logics may be given several interpretations. One would be that the derived generalizations and predictions are provisionally accepted as true. On a different interpretation, the generalizations are selected for further testing in the sense of Popper.

To simplify the picture, I shall take **CL** (Classical Logic) to be the standard of deduction and shall neglect inconsistent data as well as inconsistent theories. Even with this restriction, three problems have to be considered. The first concerns the transition from a set of empirical data to inductive generalizations. Only by taking this step, we may hope to get a grasp on the world—to understand the world and to act in it. This is handled by the logics of inductive generalization. The consequences follow from the premises either deductively, by **CL**, or inductively. The interesting consequences are obviously the inductive ones. These comprise, first and foremost, empirical generalizations. They also comprise deductive consequences of the premises and of the generalizations, including singular statements that may serve the purposes of prediction and explanation.

The second problem is that, in order to make the enterprise realistic, data have to be combined with background knowledge before moving to generalizations. I shall show that this different problem may also be solved by means of adaptive logics. The third problem, finally, is that scientists often launch conjectures that cannot be justified on *logical* grounds. We shall see that the logics of inductive generalization indicate which choices may be made. Different adaptive logics guide the introduction of the conjectures and make the process defeasible.

It is well known that induction runs into trouble if relational predicates are considered. These allow one to push the whole history of the world into the present properties of an object. So I shall restrict the attention to unary predicates. First problems have to be solved first. There will be further restrictions. By a primitive functional formula of rank 1, I shall mean an open formula that does not contain any logical symbols, sentential letters, or individual constants, and that contains only predicative letters of rank 1. Let $\mathcal{A}^{f1}$, the set of functional atoms of rank 1, comprise the primitive functional formulas of rank 1 and their negations. By a *generalization* I shall henceforth mean the universal closure of a disjunction of members of $\mathcal{A}^{f1}$, formally $\{\forall(A_1 \vee \ldots \vee A_n) \mid A_1, \ldots, A_n \in \mathcal{A}^{f1}; n \geq 1\}$, in which $\forall$ obviously denotes the universal closure of the subsequent formula. Actually, that identity would be allowed to occur in members of $\mathcal{A}^{f1}$ would not make a significant difference. The absence of sentential letters, individual constants, and quantifiers in generalizations was already justified in [3]. Incidentally, all (thus restricted) generalizations are equivalent to formulas of the form $\forall((B_1 \wedge \ldots \wedge B_n) \supset C_1)$ and hence also to formulas of the form $\forall((B_1 \wedge \ldots \wedge B_n) \supset (C_1 \vee \ldots \vee C_m))$, in which all B_i and C_i belong to $\mathcal{A}^{f1}$. For some logics introduced in subsequent sections, it would not make any difference that universal closures of other truth-functions of members of $\mathcal{A}^{f1}$ were counted as generalizations. The advantage of the present convention is that it provides a unifying framework.

When working on inductive generalization, for example on [3] and [10], I wondered why systems as simple and clarifying as the logics articulated below had not been presented a long time ago.[2] The reason is presumably that their formulation presupposes familiarity with the adaptive logic programme. Yet, the logics are extremely simple and extremely promising.

As announced, all relevant metatheoretic results are available elsewhere. The aim of the present paper is to illustrate the variety of inductive methods that may be characterized by adaptive logics, even if deductive logic is identified with **CL**. So the paper is mainly an argument for the richness of the adaptive approach, which had its effects on the bibliography. Comparison with other approaches is not possible within the confines of the present paper. Other approaches are so different that this comparison would require a separate paper.

2 Adaptive Logics

As there are many introductions to adaptive logics, for example [2] and [6], I shall be as brief as possible. An adaptive logic, **AL**, in standard format is a triple:

1. A *lower limit logic* **LLL**: a reflexive, transitive, monotonic, and compact logic that has a characteristic semantics and contains **CL** (Classical Logic).
2. A *set of abnormalities* Ω : a set of **LLL**-contingent formulas, characterized by a (possibly restricted) logical form F; or a union of such sets.
3. An *adaptive strategy*: Reliability or Minimal Abnormality.

The lower limit logic is the stable part of the adaptive logic; anything that follows from the premises by **LLL** will never be revoked. The lower limit logic is an extension of **CL** in that the standard language is extended with all classical symbols (noted with a check: $\check{\neg}$, $\check{\vee}$, ..., $\check{\exists}$, $\check{=}$). The classical symbols do not occur in the premises or in the conclusion; they serve a technical and metatheoretical function. Abnormalities are supposed to be false, 'unless and until proven otherwise'. Strategies are ways to cope with derivable disjunctions of abnormalities: an adaptive strategy picks one specific way to interpret the premises as normally as possible.[3]

If the lower limit logic is extended with an axiom that declares all abnormalities logically false, one obtains the *upper limit logic* **ULL**. If a premise set Γ does not require that any abnormalities are true, the **AL**-consequences of Γ are identical to its **ULL**-consequences.

In the expression $Dab(\Delta)$, Δ is a finite subset of Ω and $Dab(\Delta)$ denotes the *classical* disjunction of the members of Δ. $Dab(\Delta)$ is called a *Dab*-formula. $Dab(\Delta)$ is a *minimal Dab-consequence of* Γ iff $\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta)$ whereas $\Gamma \nvdash_{\mathbf{LLL}} Dab(\Delta')$ for any $\Delta' \subset \Delta$. Where $Dab(\Delta_1)$, $Dab(\Delta_2)$, ... are the minimal *Dab*-consequences of Γ, $U(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$ The set $U(\Gamma)$ comprises the

[2] In the form of a formal logic, that is. There are connections with Mill's canons. There are also connections with Reichenbach's straight rule, if restricted to general hypotheses, and with Popper's conjectures and refutations. Articulating the formal logic drastically increases precision, as we shall see.

[3] Reliability and Minimal Abnormality are the official strategies of the standard format. Most other strategies were developed to characterize consequence relations from the literature in terms of an adaptive logic. All those strategies can be reduced to Reliability or Minimal Abnormality under a translation.

abnormalities that are *unreliable* with respect to Γ. Where M is a **LLL**-model, $Ab(M)$ is the set of abnormalities verified by M.

Definition 2.1 *A* **LLL**-*model* M *of* Γ *is* reliable *iff* $Ab(M) \subseteq U(\Gamma)$.

Definition 2.2 $\Gamma \vDash_{\mathbf{AL}^r} A$ *iff* A *is verified by all reliable models of* Γ.

Definition 2.3 *A* **LLL**-*model* M *of* Γ *is* minimally abnormal *iff there is no* **LLL**-*model* M' *of* Γ *such that* $Ab(M') \subset Ab(M)$.

Definition 2.4 $\Gamma \vDash_{\mathbf{AL}^m} A$ *iff* A *is verified by all minimally abnormal models of* Γ.

The two strategies are not as different as the above treatment may suggest. A *choice set* of $\Sigma = \{\Delta_1, \Delta_2, \ldots\}$ is a set that contains an element out of each member of Σ. A *minimal choice set* of Σ is a choice set of Σ of which no proper subset is a choice set of Σ. Where $Dab(\Delta_1)$, $Dab(\Delta_2)$, ... are the minimal *Dab*-consequences of Γ, $\Phi(\Gamma)$ is the set of minimal choice sets of $\Sigma = \{\Delta_1, \Delta_2, \ldots\}$. It can be shown that a **LLL**-model M of Γ is *minimally abnormal* iff $Ab(M) \in \Phi(\Gamma)$.

Apart from a semantics, adaptive logics have a dynamic proof theory—see for example [8] for some theory. An annotated **AL**-proof consists of lines that have four elements: a line number, a formula, a justification (at most referring to preceding lines) and a *condition*. Where

$$A \qquad \Delta$$

abbreviates that A occurs in the proof as the formula of a line that has Δ as its condition, the (generic) inference rules are:

PREM If $A \in \Gamma$:
$$\begin{array}{ll} \ldots & \ldots \\ \hline A & \emptyset \end{array}$$

RU If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$:
$$\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

RC If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \check{\vee} Dab(\Theta)$
$$\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

A *stage* of a proof is a list of lines obtained by applications of the three above rules. The empty list will be considered as stage 0 of every proof. Where s is a stage, s' is an *extension* of s iff all lines that occur in s occur in the same order in s'. A (dynamic) proof is a chain of stages. Here comes a peculiarity required by the Minimal Abnormality strategy. Normally, the extension of a stage is obtained by appending lines. This is not required here. The added lines may be inserted, provided that the justification of every line refers only to preceding lines. A line inserted between lines 4 and 5 may, for example, be numbered 4.1 (or the lines may be renumbered).

That A is derivable on the condition Δ may be interpreted as follows: it follows from the premise set that A or one of the members of Δ is true. As the members of Δ, which are abnormalities, are supposed to be false, A is considered as derived, unless and until it shows that the supposition cannot be upheld. The precise meaning of "cannot be upheld" depends on the strategy, which determines the marking definition (see below) and hence determines which lines are marked at a stage. If a line is marked at a stage, its formula is considered as not derived at that stage.

$Dab(\Delta)$ is a *minimal Dab-formula* at stage s of an **AL**-proof iff $Dab(\Delta)$ has been derived at that stage on the condition $\emptyset$ whereas there is no $\Delta' \subset \Delta$ for which $Dab(\Delta')$ has been derived on the condition $\emptyset$. Where $Dab(\Delta_1), \ldots, Dab(\Delta_n)$ are the minimal Dab-formulas at stage s of a proof from Γ, $U_s(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n$ and $\Phi_s(\Gamma)$ is the set of minimal choice sets of $\{\Delta_1, \ldots, \Delta_n\}$.

Definition 2.5 *Marking for Reliability: Line l is marked at stage s iff, where Δ is its condition, $\Delta \cap U_s(\Gamma) \neq \emptyset$.*

Definition 2.6 *Marking for Minimal Abnormality: Line l is marked at stage s iff, where A is derived on the condition Δ on line l, (i) there is no $\varphi \in \Phi_s(\Gamma)$ such that $\varphi \cap \Delta = \emptyset$, or (ii) for some $\varphi \in \Phi_s(\Gamma)$, there is no line on which A is derived on a condition Θ for which $\varphi \cap \Theta = \emptyset$.*

This reads more easily: where A is derived on the condition Δ on line l, line l is *unmarked* at stage s iff (i) there is a $\varphi \in \Phi_s(\Gamma)$ for which $\varphi \cap \Delta = \emptyset$ and (ii) for every $\varphi \in \Phi_s(\Gamma)$, there is a line at which A is derived on a condition Θ for which $\varphi \cap \Theta = \emptyset$.

A formula A is *derived at stage* s of a proof from Γ iff it is the formula of a line that is unmarked at that stage. Marks may come and go as the proof proceeds. So one also wants to define a stable notion of derivability, which is called *final derivability.*

Definition 2.7 *A is* finally derived *from Γ on line l of a stage s iff (i) A is the second element of line l, (ii) line l is not marked at stage s, and (iii) every extension of the stage in which line l is marked may be further extended in such a way that line l is unmarked.*

Definition 2.8 $\Gamma \vdash_{\mathbf{AL}} A$ *(A is* finally **AL**-derivable *from Γ) iff A is finally derived on a line of a proof from Γ.*

In Definition 2.7, s may be taken to be a *finite stage* for both strategies. For the Reliability strategy, the definition may moreover be taken to refer to *finite extensions* only. For Minimal Abnormality the definition should be required to refer to finite as well as to infinite extensions, as was shown in [1, p. 479].

An intuitive notion behind final derivability is the existence of a proof that is *stable with respect to an unmarked line l*: A is derived on line l and line l is unmarked in the proof and in all its extensions. However, for some **AL**, Γ, and A, only an infinite proof from Γ in which A is the formula of a line l is stable with respect to line l. Definition 2.7 has an attractive game-theoretic interpretation—see especially [8].

3 Mere Falsification

A generalization is compatible with a set of data just in case the data do not falsify it. Let us try out that crude idea, which agrees with the hypothetico-deductive approach.

The generalizations that are inductively derived from a set of data should be *jointly* compatible with the data. The logic of compatibility—see [11]—provides us with the set of all statements compatible with Γ. However, to select a set of jointly compatible statements *in a justified way* seems hopeless. For any statement A that does not deductively follow from the premises, there is a set of statements Δ such that the members of Δ are jointly compatible with the premises whereas the members of $\Delta \cup \{A\}$ are not. However, it turns out possible to use joint compatibility as a criterion provided one considers only generalizations as restricted in Section 1.

So while the unconditional rule will take care of **CL**-consequences, the conditional rule will allow one to introduce a generalization on a condition, which will be (the singleton comprising) the negation of the generalization. In other words, the abnormalities are negations of generalizations. Why are negations of generalizations called abnormalities? There is on the one hand a technical justification for doing so: they allow us to derive the right generalizations. There is also a philosophical justification: in the present context, negations of generalizations are considered as false until and unless proven otherwise, viz. until an unless the generalization is falsified. There is also a different philosophical justification. Induction is connected to the presupposition that the world is as uniform as allowed by the data—see already [13]. The negation of a generalization expresses a lack of uniformity.

Let us call this adaptive logic $\mathbf{LI}^r$ if Reliability is chosen as the strategy. Here is the start of a very simple proof from $\Gamma_1 = \{(Pa \wedge Pb) \wedge Pc, Rb \vee \neg Qb, Rb \supset \neg Pb, (Sa \wedge Sb) \wedge Qa\}$. Some **CL**-consequences are derived and two generalizations are introduced.

1	$(Pa \wedge Pb) \wedge Pc$	premise	$\emptyset$
2	$Rb \vee \neg Qb$	premise	$\emptyset$
3	$Rb \supset \neg Pb$	premise	$\emptyset$
4	$(Sa \wedge Sb) \wedge Qa$	premise	$\emptyset$
5	Pa	1; RU	$\emptyset$
6	Pb	1; RU	$\emptyset$
7	Qa	4; RU	$\emptyset$
8	Sa	4; RU	$\emptyset$
9	Sb	4; RU	$\emptyset$
10	$\forall x(Px \supset Sx)$	RC	$\{\neg\forall x(Px \supset Sx)\}$
11	$\forall x(Px \supset Qx)$	RC	$\{\neg\forall x(Px \supset Qx)\}$

The two generalizations are considered as 'conditionally' true, as true until falsified. The sole member of $\{\neg\forall x(Px \supset Sx)\}$ has to be false in order for $\forall x(Px \supset Sx)$ to be derivable and similarly for line 11. Conditionally derived formulas may obviously be combined by RU. Here is an example:

12	$\forall x(Px \supset (Qx \wedge Sx))$	10, 11; RU	$\{\neg\forall x(Px \supset Sx), \neg\forall x(Px \supset Qx)\}$

The interpretation of the condition of line 12 is obviously that $\forall x(Px \supset (Qx \wedge Sx))$ should be considered as not derived if $\neg\forall x(Px \supset Sx)$ or $\neg\forall x(Px \supset$

Qx) turns out to be true. Actually, it is not difficult to see that $\neg\forall x(Px \supset Qx)$ is indeed derivable from the premises.

13	$\neg Rb$	3, 6; RU	$\emptyset$
14	$\neg Qb$	2, 13; RU	$\emptyset$
15	$\neg\forall x(Px \supset Qx)$	6, 14; RU	$\emptyset$

So lines 11 and 12 have to be *marked* and their formulas are considered as not inductively derived from the premises. I repeat lines 10–15 with the correct marks at stage 15:

10	$\forall x(Px \supset Sx)$	RC	$\{\neg\forall x(Px \supset Sx)\}$	
11	$\forall x(Px \supset Qx)$	RC	$\{\neg\forall x(Px \supset Qx)\}$	$\checkmark^{15}$
12	$\forall x(Px \supset (Qx \wedge Sx))$	10, 11; RU	$\{\neg\forall x(Px \supset Sx), \neg\forall x(Px \supset Qx)\}$	$\checkmark^{15}$
13	$\neg Rb$	3, 6; RU	$\emptyset$	
14	$\neg Qb$	2, 13; RU	$\emptyset$	
15	$\neg\forall x(Px \supset Qx)$	6, 14; RU	$\emptyset$	

Derived generalizations obviously entail predictions by RU, as in line 17.

16	Pc	1; RU	$\emptyset$
17	Sc	16, 10; RU	$\{\neg\forall x(Px \supset Sx)\}$

At this point I have to report about a fascinating phenomenon. Although we started from a simple hypothetico-deductive approach, based on falsification, it turns out that some abnormalities are connected. Not only falsification, but also the connection between abnormalities, which comes to joint falsification, will prevent certain generalizations from being adaptively derivable. I now illustrate this. In the atoms derived so far, there are objects known to be P, but none known to be $\neg P$, and objects known to be $\neg R$, but none known to be R. So it seems attractive to introduce two generalizations expressing this. I shall first present the extension of the proof and then comment upon it.

18	$\forall x Px$	RC	$\{\neg\forall x Px\}$	
19	$\forall x \neg Rx$	RC	$\{\neg\forall x \neg Rx\}$	$\checkmark^{22}$
20	$Ra \vee \neg Ra$	RU	$\emptyset$	
21	$Ra \vee (Qa \wedge \neg Ra)$	20, 7; RU	$\emptyset$	
22	$\neg\forall x \neg Rx \vee \neg\forall x(Qx \supset Rx)$	21; RU	$\emptyset$	

The formula unconditionally derived at line 22 is a minimal *Dab*-formula at stage 22 (and actually also a minimal *Dab*-consequence of the premise set). So line 19 is marked at this stage and is actually marked in every extension of the stage. The generalization $\forall x Px$, derived at line 18, is finally derived. There is no minimal *Dab*-consequence of the premises of which $\neg\forall x Px$ is a disjunct.

The phenomenon is fascinating because the adaptive approach reveals that connected abnormalities prevent one from deriving the related generalizations. Although we started from a hypothetico-deductive approach that deems a generalization non-derivable in case it is falsified, we arrived at the insight that a generalization may not be derivable because it belongs to a minimal (finite) set of generalizations one of which is bound to be falsified by the data. I don't think this insight is very deep. Yet, much of the literature that sees the derivation of generalizations along these lines, simply missed this point.

This deserves a further comment. Apparently joint compatibility is not seen as a suitable means to distinguish between generalizations that may sensibly be derived from a set of data and those that may not sensibly be so derived. The reason is obviously that every generalization belongs to a set of generalizations that is incompatible with the data. However, $\mathbf{LI}^r$ solves this predicament. It allows one to derive the generalizations that do *not* belong to a *minimal* set of generalizations that are jointly incompatible with the data. For this reason $\forall x \neg Rx$ is not derivable from the considered data, but $\forall x Px$ is. Indeed, $\neg\forall x Px$ is not a disjunct of a minimal *Dab*-consequence of the data—note that a *Dab*-formula is equivalent to the negation of a conjunction of generalizations. It is precisely by invoking *minimality* that the above criterion is made to do its job.[4] So, as I suggested, the adaptive approach gets it right from the beginning.

The role of connected abnormalities may also be illustrated by considering a predicate that does not occur in the premises. T is such a predicate, and one might introduce $\forall x(Qx \supset Tx)$ to see what becomes of it. Not much, as the following extension of the proof shows.

23	$\forall x(Qx \supset Tx)$	RC	$\{\neg\forall x(Qx \supset Tx)\}$	$\checkmark^{25}$
24	$\forall x(Qx \supset \neg Tx)$	RC	$\{\neg\forall x(Qx \supset \neg Tx)\}$	$\checkmark^{25}$
25	$\neg\forall x(Qx \supset Tx) \vee \neg\forall x(Qx \supset \neg Tx)$	7; RU	$\emptyset$	

Obviously, the premises 1–4 do not contradict $\forall x(Qx \supset Tx)$. However, they contradict $\forall x(Qx \supset Tx) \wedge \forall x(Qx \supset \neg Tx)$, which is noted on line 25 and causes lines 23 and 24 to be marked. Incidentally, suppose that the marking definition would causes a line to be marked only if an element of its condition is *derived* unconditionally. On such a definition, lines 23 and 24 would both be unmarked and so would be a line at which we would derive $Ta \wedge \neg Ta$ from 1, 23, and 24. So triviality would result.

Allow me to insert a short intermission at this point. By applying RU, viz. Addition, one may derive

$$\forall x(Qx \supset Tx) \vee \forall x(Qx \supset \neg Tx)$$

on two different conditions: from line 23 on the condition $\{\neg\forall x(Qx \supset Tx)\}$ and from line 24 on the condition $\{\neg\forall x(Qx \supset \neg Tx)\}$. The two lines on which the disjunction would be so derived would still be marked on the Reliability strategy. However, once both occur, they are unmarked if Minimal Abnormality is the strategy, so for the logic $\mathbf{LI}^m$. This illustrates that Minimal Abnormality leads to some more consequences than Reliability.

At this point, I can also clarify some of the restrictions on generalizations—these are actually introduced by restrictions on the abnormalities. Suppose that one allowed to introduce the generalization $\forall x((Qx \vee \neg Qx) \supset \neg Sc)$ on the condition $\{\neg\forall x((Qx \vee \neg Qx) \supset \neg Sc)\}$ in the preceding proof. From line 1 follows

$$\neg\forall x(Px \supset Sx) \vee \neg\forall x((Qx \vee \neg Qx) \supset \neg Sc)$$

by RU. This would cause the line on which $\forall x((Qx \vee \neg Qx) \supset \neg Sc)$ is derived to be marked. However, it would also cause line 10 to be marked. It is not

[4]The compactness of **CL** warrants that every *minimal* set of formulas (any formulas that is), the members of which are jointly incompatible with a premise set (any premise set), is finite.

difficult to see that, by allowing such 'generalizations', no formula derived on a non-empty condition would be finally derived. Similar troubles arise if it were allowed to introduce such hypotheses as $\forall x((Qx \vee \neg Qx) \supset (\exists y)(Py \wedge \neg Sy))$.

Before leaving the matter, let me add two comments on the logics $\mathbf{LI}^r$ and $\mathbf{LI}^m$. First, formulating their semantics is obvious in view of Section 2. The second comment is that some readers may argue that the dynamics of the proofs may be avoided in view of the restrictions on the premises and on the conclusions. This is correct. However, the resulting proofs are not the usual static proofs of Tarski logics, but a complication of dynamic proofs. More importantly, the present logic is a very restricted one. For example, it does not take background knowledge into account. As soon as some of the restrictions are eliminated, we obtain a consequence relation for which there is no positive test and which requires dynamic proofs.

4 Two Stronger Alternatives

If uniformity is identified with the truth of *every* generalization, as in $\mathbf{LI}^r$ and $\mathbf{LI}^m$, no possible world is completely uniform. Uniformity may be interpreted differently: a completely uniform world is one in which all objects have the same properties. Put differently, if something has a property, then everything has this property. So $\exists xPx \supset \forall xPx$ and, in general, $\exists A \supset \forall A$ in which A is a disjunction of members of $\mathcal{A}^{f1}$. Some worlds are completely uniform in this sense, although not our world.

The adaptive logics based on this idea will be called $\mathbf{IL}^r$ and $\mathbf{IL}^m$. An abnormality, viz. a formula expressing a lack of uniformity, states that something has a property and something else does not. It is a formula of the form $\exists(A_1 \vee \ldots \vee A_n) \wedge \exists\neg(A_1 \vee \ldots \vee A_n)$ for $A_1, \ldots, A_n \in \mathcal{A}^{f1}$ and $n \geq 1$. Here is a little proof from $\{Pa\}$.

1	Pa	premise	$\emptyset$
2	$\forall xPx$	1; RC	$\{\exists Px \wedge \exists\neg Px\}$

Several simple but interesting observations may be made. From the premise Pa, $\exists xPx$ is $\mathbf{CL}$-derivable. As $\forall xPx \vee \exists x\neg Px$ is a $\mathbf{CL}$-theorem, either $\forall xPx$ is true or the abnormality $\exists Px \wedge \exists\neg Px$ is true. In other words, if one presupposes abnormalities to be false unless and until proven otherwise, one may derive $\forall xPx$ from Pa, unless and until some object is shown not to have the property P. Next, if Pa is a premise, $\exists Px \wedge \exists\neg Px$ is false just in case $\exists\neg Px$ is false, and this formula is $\mathbf{CL}$-equivalent to the negation of $\forall xPx$. This provides a nice way to compare $\mathbf{IL}^r$ with $\mathbf{LI}^r$. The logic $\mathbf{LI}^r$ enables one to introduce $\forall xPx$ on the condition $\{\neg\forall xPx\}$ because $\forall xPx \vee \neg\forall xPx$ is a $\mathbf{CL}$-theorem. $\mathbf{IL}^r$ seems more demanding. In order to introduce $\forall xPx$, one needs an instance of it, or at least $\exists xPx$. This seems a natural requirement; one does not want to introduce a generalization unless one knows that it has at least one instance.

While $\mathbf{IL}^r$ is very close to $\mathbf{LI}^r$, it is also richer than $\mathbf{LI}^r$. This is easily seen if the previous proof is continued as follows—$!A$ abbreviates $\exists A \wedge \exists\neg A$ to fit the proof on the page.

3	$(Pa \supset Qa) \vee (Pa \supset \neg Qa)$	1; RU	$\emptyset$
4	$\forall x(Px \supset Qx) \vee \forall x(Px \supset \neg Qx)$	3; RC	$\{!(\neg Px \vee Qx), !(\neg Px \vee \neg Qx)\}$

Although we do now know anything about the Q-hood of objects that are P, we are presupposing uniformity. So we presuppose that the objects that are P are either all Q or all not Q. In view of this, it is desirable that $\forall x(Px \supset Qx) \vee \forall x(Px \supset \neg Qx)$ is finally derivable from $\{Pa\}$. And indeed it is. As the premise set is normal (with respect to $\mathbf{IL}^r$), no abnormality is **CL**-derivable from it and line 4 will not be marked in any extension of the proof. This is a gain over $\mathbf{LI}^r$. In a $\mathbf{LI}^r$-proof from $\{Pa\}$, the (reformulated) condition of line 4 is $\{\exists x\neg(\neg Px \vee Qx), \exists x\neg(\neg Px \vee \neg Qx)\}$ and the disjunction of the two members of the condition is **CL**-derivable from Pa, whence line 4 is marked in the $\mathbf{LI}^r$-proof.

The logic $\mathbf{IL}^r$ is also richer than $\mathbf{LI}^r$ in other respects. Here is a nice example, provided to me by Mathieu Beirlaen. Consider $\Gamma_2 = \{Pa, \neg Pb \vee Qb\}$. In $\mathbf{LI}^r$ no generalization is derivable from Γ_2. Indeed, Pa entails the abnormality $\neg\forall x\neg Px$. The second premise informs one that either the abnormality $\neg\forall xPx$ or $\neg\forall x\neg Qx$ obtains. Note that both choices are on a par. The proof goes as follows.

1	Pa	premise	$\emptyset$	
2	$\neg Pb \vee Qb$	premise	$\emptyset$	
3	$\forall xPx$	1; RC	$\{\neg\forall xPx\}$	$\checkmark^4$
4	$\neg\forall xPx \vee \neg\forall x\neg Qx$	1, 2; RU	$\emptyset$	

So line 3 is marked in every extension of the proof from Γ because $\exists x\neg Px \in U_s(\Gamma_2)$. The situation in $\mathbf{IL}^r$ is completely different.

1	Pa	premise	$\emptyset$
2	$\neg Pb \vee Qb$	premise	$\emptyset$
3	$\forall xPx$	1; RC	$\{\exists xPx \wedge \exists x\neg Px\}$
4	$\forall xQx$	3, 2; RC	$\{\exists xPx \wedge \exists x\neg Px, \exists x\neg Qx \wedge \exists x\neg\neg Qx\}$

With respect to $\mathbf{IL}^r$, Γ_2 is normal (no *Dab*-formula is derivable from it). So both lines 3 and 4 are unmarked in all extensions of the proof.

The variant $\mathbf{IL}^m$ is similar except that it has Minimal Abnormality as its strategy and is slightly stronger than $\mathbf{IL}^r$.

Consider $\mathbf{IL}^r$-proof from $\Gamma_3 = \{Pa, Qa, \neg Qb, \neg Pc\}$.

1	Pa	premise	$\emptyset$	
2	Qa	premise	$\emptyset$	
3	$\neg Qb$	premise	$\emptyset$	
4	$\neg Pc$	premise	$\emptyset$	
5	$\forall x(Px \supset Qx)$	2; RC	$\{!(\neg Px \vee Qx)\}$	$\checkmark^7$
6	$(Pb \wedge \neg Qb) \vee (\neg Pb \wedge \neg Qb)$	3; RU	$\emptyset$	
7	$!(\neg Px \vee Qx) \vee !(Px \vee Qx)$	2, 6; RC	$\emptyset$	

As $\Gamma_3 \nvdash_{\mathbf{CL}} !(Px \vee Qx)$, line 5 is marked in all extensions of the proof. Even $\mathbf{IL}^m$ assigns only some disjuncts of generalizations as final consequences to Γ_3. Presumably some would like to derive $\forall x(Px \equiv Qx)$ from it. This does not follow by $\mathbf{IL}^r$, but it follows if we slightly modify the adaptive logic.

The instances that are required to occur in the $\mathbf{IL}^m$-abnormalities are instances in the sense in which logicians use the term, for example $Pd \supset Qd$ is an instance of $Px \supset Qx$. When philosophers of science talk about inductive generalization, they use the phrases *positive* instance and *negative* instance. By a positive instance of $\forall x(Px \supset Qx)$ they mean an instance of $Px \wedge Qx$ and by a negative instance of $\forall x(Px \supset Qx)$ they mean an instance of $Px \wedge \neg Qx$. This suggests that we consider adaptive logics that have such abnormalities as $\exists x(Px \wedge Qx) \wedge \exists x(Px \wedge \neg Qx)$—in words: there is a positive as well as a negative instance of $\forall x(Px \supset Qx)$.

One way to do this systematically is by defining the set of abnormalities as $\{\exists(A_1 \wedge \ldots \wedge A_n \wedge A_0) \wedge \exists(A_1 \wedge \ldots \wedge A_n \wedge \neg A_0) \mid A_0, A_1, \ldots, A_n \in \mathcal{A}^{f1}; n \geq 0\}$—if $n = 0$, the formula reduces to $\exists A_0 \wedge \exists \neg A_0$. As the abnormalities are long, I shall abbreviate $\exists(A_1 \wedge \ldots \wedge A_n \wedge A_0) \wedge \exists(A_1 \wedge \ldots \wedge A_n \wedge \neg A_0)$ in proofs as $A_1 \wedge \ldots \wedge A_n \wedge \pm A_0$.[5]

This approach is sufficiently general because every generalization, as restricted in Section 1, is **CL**-equivalent to a conjunction of formulas of the form $\forall((A_1 \wedge \ldots \wedge A_n) \supset A_0)$ in which the metavariables denote members of $\mathcal{A}^{f1}$. So a generalization G will be derivable just in case the formulas are derivable that have the specified form and are **CL**-entailed by G. This is precisely as we want it. Note that $\forall((A_1 \wedge \ldots \wedge A_n) \supset A_0)$ is equivalent to $\forall(\neg A_1 \vee \ldots \vee \neg A_n \vee A_0)$ and that the $n + 1$ abnormalities correspond to this formula. Example: $\forall x((Px \wedge Qx) \supset Rx)$, $\forall x((Px \wedge \neg Rx) \supset \neg Qx)$, and $\forall x((\neg Rx \wedge Qx) \supset \neg Px)$ are equivalent but generate three different abnormalities—only their first conjunct is non-equivalent. In other words, the generalization $\forall x(\neg Px \vee \neg Qx \vee Rx)$ can be derived on three different conditions.

Let us call the so obtained logics $\mathbf{G}^r$ and $\mathbf{G}^m$. Their lower limit is obviously **CL**, their set of abnormalities is the one defined two paragraphs ago, and their strategies are respectively Reliability and Minimal Abnormality. Let us now return to the premise set Γ_3. I skip the premises in the following proof.

5	$\forall x(Px \supset Qx)$	1, 2; RC	$\{Px \wedge \pm Qx\}$
6	$\forall x(Qx \supset Px)$	1, 2; RC	$\{Qx \wedge \pm Px\}$
7	$\forall x(Qx \equiv Px)$	5, 6; RU	$\{Px \wedge \pm Qx, Qx \wedge \pm Px\}$

Line 5 will not be marked in any extension of this proof because the only minimal *Dab*-consequences of Γ_3 are $\exists xPx \wedge \exists x\neg Px$ and $\exists xQx \wedge \exists x\neg Qx$. Obviously the proof is also a correct $\mathbf{G}^m$-proof and no line is marked in it or will be marked in any extension.

The reader may think that $\forall xQx \vee \forall x\neg Qx$ is a final $\mathbf{G}^r$-consequence of $\{Pa, \neg Pb\}$. This is mistaken because these premises **CL**-entail the *Dab*-formula $\pm Qx \vee (Qx \wedge \pm Px) \vee (\neg Qx \wedge \pm Px)$. This brings us right to the topic of the following section.

5 Combined Adaptive Logics

As spelling out the proof theory would lead us too far, I shall define the combined adaptive logics in terms of the consequence sets of simple (that is, uncombined) adaptive logics. The combined logics rely on the Popperian idea that more general hypotheses should be given precedence over less general ones. So if $\forall xPx$ and $\forall x(Qx \supset Rx)$ are jointly incompatible with the data, the combined logics will (given the right circumstances) deliver $\forall xPx$ as a consequence.

[5]The definition of the abnormalities does not require that the A_i are different from each other, and indeed there is no need to require so.

For each logic from the previous section, we first split the sets of abnormality, Ω, into subsets of different degrees. The degree of an abnormality is one less than the number of atoms that occur in (one conjunct of) it. Thus the degree of the **G**-abnormality $\exists x(A_1 \wedge \ldots \wedge A_n \wedge A_0) \wedge \exists x(A_1 \wedge \ldots \wedge A_n \wedge \neg A_0)$ is n. Splitting Ω thus, we obtain sets $\Omega^0, \Omega^1, \ldots$. Next we define, for each $i \geq 0$, $\Omega^{(i)} = \Omega^0 \cup \Omega^1 \cup \ldots \cup \Omega^i$. Finally, we define, for each of the logics from the previous section, an infinity of logics which are just like the original, except that their set of abnormalities is Ω^i, respectively $\Omega^{(i)}$. This gives us logics called $\mathbf{LI}^r_i$, $\mathbf{LI}^m_i$, $\mathbf{LI}^r_{(i)}$, $\mathbf{LI}^m_{(i)}$, $\mathbf{IL}^r_i$, and so on. From these logics, the combined adaptive logics are defined. Let us start with the **H**-group, the H referring to the hierarchical character of the logic.

$$Cn_{\mathbf{HLI}^r}(\Gamma) = Cn_{\mathbf{CL}}(Cn_{\mathbf{LI}^r_{(0)}}(\Gamma) \cup Cn_{\mathbf{LI}^r_{(1)}}(\Gamma) \cup \ldots) \tag{1}$$

The definitions for the **IL**-family and the **G**-family are similar—replace "LI" by "IL", respectively "G" in (1). For the logics that follow the Minimal Abnormality strategy, replace the superscript r by m everywhere. Note that the definiens is a union of consequence sets closed under the lower limit logic **CL**.

Some readers might get scared by (1). The logic $\mathbf{HLI}^r$ is defined in terms of an infinity of logics. This, however, is not a problem. It is not a theoretical problem because all those logics are well-defined and so is $\mathbf{HLI}^r$. It is not a practical problem either because the premise sets to which we want to apply $\mathbf{HLI}^r$ is forcibly a finite set of singular data and people applying $\mathbf{HLI}^r$ will only be interested in hypotheses built from predicates that occur in the data. So every application will require that at most a finite number of $\mathbf{LI}^r_{(i)}$ logics are invoked.

Next, we come to the **C**-group.

$$Cn_{\mathbf{CLI}^r}(\Gamma) = \ldots(Cn_{\mathbf{LI}^r_{(2)}}(Cn_{\mathbf{LI}^r_{(1)}}(Cn_{\mathbf{LI}^r_{(0)}}(\Gamma))))\ldots \tag{2}$$

Here we have a superposition of consequence sets. This means that the *Dab*-formulas of level 1 are defined in terms of $Cn_{\mathbf{LI}^r_{(0)}}(\Gamma)$ and not in terms of Γ. Applying the same type of combination to the **IL**-family and to the **G**-family is an obvious task, and so is defining the corresponding logics that have Minimal Abnormality as their strategy.

Finally, here is the definition of a logic from the **S**-group.

$$Cn_{\mathbf{SLI}^r}(\Gamma) = \ldots(Cn_{\mathbf{LI}^r_2}(Cn_{\mathbf{LI}^r_1}(Cn_{\mathbf{LI}^r_0}(\Gamma))))\ldots \tag{3}$$

The basic difference with the **C**-group is that the sets of abnormalities are kept apart. In other words, if an abnormality of level n is not unreliable, then it cannot be unreliable at any higher level.

It is provable for each of the combined logics that the consequence set of Γ is non-trivial whenever Γ is not inconsistent. The proof theory of the combined logics is not spelled out here, but its fascinating feature is that it is not more complex than the proof theory of the simple logics. This holds for the construction of proofs as well as for the criteria that warrant final derivability. All aforementioned logics are listed in the following table.

	LI-family		**IL**-family		**G**-family	
simple	$\mathbf{LI}^r$	$\mathbf{LI}^m$	$\mathbf{IL}^r$	$\mathbf{IL}^m$	$\mathbf{G}^r$	$\mathbf{G}^m$
H-group	$\mathbf{HLI}^r$	$\mathbf{HLI}^m$	$\mathbf{HIL}^r$	$\mathbf{HIL}^m$	$\mathbf{HG}^r$	$\mathbf{HG}^m$
C-group	$\mathbf{CLI}^r$	$\mathbf{CLI}^m$	$\mathbf{CIL}^r$	$\mathbf{CIL}^m$	$\mathbf{CG}^r$	$\mathbf{CG}^m$
S-group	$\mathbf{SLI}^r$	$\mathbf{SLI}^m$	$\mathbf{SIL}^r$	$\mathbf{SIL}^m$	$\mathbf{SG}^r$	$\mathbf{SG}^m$

6 Making it More Realistic

Two features that make the enterprise more realistic will be discussed: taking background knowledge into account and allowing the researcher to launch conjectures that are not justified by logical considerations. I shall also briefly comment upon the research-guiding aspects of the logics.

The easiest way to handle background knowledge is by adding it to the available data, in other words to the premise set. Three problems lurk around the corner though. The first is that separate pieces of the background knowledge may be inconsistent, the second that pieces of background knowledge may be jointly inconsistent, the third that background knowledge may be falsified by the data. As this paper is written from a classical outlook, I shall disregard the first problem. The third problem obviously cannot be neglected even within a fully classical framework—no classical logician would locate anything deviant in Popper's work. The trouble is that Popper's advice to discard falsified theories is too simple. For more than thirty years—I mean [15]—everyone who can read and has access to the literature should know that scientists often continue to reason from falsified theories. They typically consider the falsifications as problems but keep relying on other consequences of the falsified theory. In other words, discarding falsified theories is not always a good advice because it would leave one without any information about the domain. The third problem will turn out to be solved by the way in which I shall handle falsification.

If falsification occurs, two cases need to be distinguished: the one in which we discard a falsified theory altogether and the one in which we only discard the falsified consequences of the theory. Note that a falsification may only be discovered after some reasoning. So we are in defeasible waters here. A further distinction has to be made. Part of our background knowledge consists of separate statements whereas other parts are theories—for present purposes, any non-singleton set of statements will be considered to be a theory.

The picture is not complete yet. That the data falsify a piece of background knowledge is the simple case. More often, however, the data falsify a set of background knowledge without falsifying any single theory or any single separate statement in the set. In such cases, it is important to take into account that elements of our background knowledge may differ in plausibility. One will reject the piece of background knowledge that is least plausible anyway. So we need plausibilities or priorities or preferences. Incidentally, this makes the matter even more defeasible.

Here is a way to handle all this adaptively. Let both $\Diamond$ and $\diamond$ be defined as **K**-possibility;[6] they have the same meaning, but behave differently in the adaptive logic. Let $\Diamond^i$ denote a sequence of i symbols $\Diamond$ and similarly for $\diamond^i$.

[6] I refer to a predicative version of **K**, spelled out in [9] but easily reconstructed from, for example [12] or [4].

Next, define, for every $i \geq 1$,

$$\Omega^i = \{\exists(\lozenge^i A \wedge \neg A) \mid A \in \mathcal{A}\} \cup \{\exists(\diamond^i B \wedge \neg B) \mid B \in \mathcal{W}\},$$

in which $\mathcal{A}$ is the set of atoms (primitive formulas and their negations) and $\mathcal{W}$ the set of closed formulas. For every $i \geq 1$, define the adaptive logic $\mathbf{K}^r_i$ as the triple composed of (i) $\mathbf{K}$, (ii) Ω^i, and (iii) Reliability. Finally, define a combined adaptive logic

$$Cn_{\mathbf{K}^r}(\Gamma) = \ldots (Cn_{\mathbf{K}^r_3}(Cn_{\mathbf{K}^r_2}(Cn_{\mathbf{K}^r_1}(\Gamma)))) \ldots \tag{4}$$

and define the combined logic $\mathbf{K}^m$ analogously.

This framework enables us to express the prioritized background knowledge in the object language. If, in the case of falsification, a piece of background knowledge A is to be discarded altogether, it is introduced in the premise set as $\diamond^i A$, in which i is smaller as A's priority is higher. If, in the case of falsification, only the falsified consequences of A have to be discarded, the piece of background knowledge A is introduced in the premise set as $\lozenge^i A$. In the case of a theory, every conjunction A of finitely many 'axioms' of the theory is so introduced as a premise.

Applying $\mathbf{K}^r$ or $\mathbf{K}^m$ to the premise set, which now comprises the data as well as the prioritized background knowledge, will have the desired effect. If a piece of background knowledge A is falsified by the data, it is discarded in case it occurs in the premises as $\diamond^i A$, for some i, and its falsified consequences are discarded in case it occurs in the premises as $\lozenge^i A$. Pieces of background knowledge, or consequences of them, that are not discarded are retained as a non-prioritized statements. The proof theory is very simple in this respect. From $\lozenge^i A$ follows $A \vee (\neg A \wedge \lozenge^i A)$ by $\mathbf{CL}$. So if $\lozenge^i A$ occurs in the proof on the condition Δ, A may be derived from it on the condition $\Delta \cup \{\neg A \wedge \lozenge^i A\}$. Moreover, all this proceeds in a prioritized way, as appears from (4). So a piece of background knowledge of priority i will be 'judged' by the data together with the retained pieces of background knowledge with priority $j < i$. The consequences obtained by $\mathbf{K}^r$ or $\mathbf{K}^m$ from the premises will then function as the premises of a logic of inductive generalization. If the latter is combined itself, this may look like a lot of combinations. So I better repeat that the proofs of such combinations are not more complex in principle than the proofs of simple adaptive logics. It is moreover worth spelling out that, in the dynamic proofs, all simple adaptive logics that play a role in the combination may be applied in any order. The effect of the combination is taken care of by the marking definition: marking proceeds in view of the different combining logics in a certain order.

The second aspect in which the procedure should be made more realistic is by allowing the researcher to introduce conjectures. Ampliative consequences derived by a logic of inductive generalization are justified in view of a logical rationale, which is slightly different for every such logic. This rationale is fixed by the choice of the abnormalities and possibly by ordering them in several sets and by the role these sets play in the combined logic. The derived generalizations are conjectures in the sense that the logical rationale is not deduction—the generalizations may be overruled by future data, the logic is defeasible. There is, however, a very different kind of conjectures, viz. those for which there is at best a non-logical justification.

Suppose that, at some stage of a proof, $A \vee B \vee C$ is a minimal *Dab*-formula (at that stage), with A, B, and C abnormalities of the right kind. Each of the three abnormalities may prevent certain generalizations from being derivable. At this point, the researcher may have a good reason to introduce $\neg A$ as a new premise. In doing so, the *Dab*-formula is reduced to $B \vee C$ and more generalizations become derivable.

Obviously, A may turn out to be derivable from the data after all, or may be derivable when new data are obtained. So $\neg A$ should not be introduced as a certainty, but as a statement that is assigned a certain plausibility but may be defeated by future experience. The technical handling of this is obvious. We just introduce $\neg A$ with the required priority. In other words we introduce the premise $\diamond^i \neg A$, or $\Diamond^i \neg A$, with the suitable i and handle it in the same way as we handled a piece of background knowledge. So we do not need any new logic in this connection.[7]

My last point for this section is that all logics of inductive generalization have a strong research guiding flavour. Consider again the case where $A \vee B \vee C$ is a minimal *Dab*-formula at that the present stage. As we learn from Wiśniewski's erotetic logic, see for example [16, 17], the disjunction $A \vee B \vee C$ invokes the question $?\{A, B, C\}$, in words: Which of A, B, and C is the case? A full answer to this question might be A (or B or C), but even $\neg A$ (or $\neg B$ or $\neg C$) would partially answer the question in that it reduces the question to a simpler one.[8] Note that the full answer A frees both B and C, at least as far as the present *Dab*-formula is concerned. Obtaining the answer A will have the effect that certain generalizations, the derivation of which was blocked by B or C, will become derivable. Note that, in the present context, A is obtained by new experience which will falsify a certain generalization. The extended experience will provide a logical reason to derive further generalizations. Note that, here too, mere conjectures may interfere to suggest more specific research.

7 In Conclusion

Quite some ground was covered. For some parts, I had to refer to other papers. It is to be hoped that this paper will attract other researchers to continue the quest, either within the adaptive logic program or within a different one. In this way and by interaction with philosophers of science, we will eventually arrive at set of methodological theories that are formulated in a precise way and precisely for this reason may be subjected to criticism.

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[7] Still, other adaptive approaches are available as may be seen from Section 8 of [7].

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